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14. ABSTRACT

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Precipitation of trapped relativistic electrons by amplified whistler waves in the magnetosphere

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I. INTRODUCTION

In the magnetosphere, energetic electrons in the radiation belts are trapped by the Earth's dipole magnetic field and undergo a bouncing motion about the geomagnetic equator. Very energetic electrons (those in the MeV levels) have a strong impact on passing satellite systems. Satellites are designed to survive a certain amount of radiation (ionizing) dose accumulated during their lifetimes. Unexpected enhancement of the radiation fluxes caused by, for example, very strong solar storms, will significantly increase the total radiation dose to the satellites. Consequently, the radiation damage on active electronics and detectors of satellite systems will accumulate much faster than designed for. As the damage exceeds a threshold level, satellite systems become incapable of performing their mission. Therefore, mitigation of unexpected radiation enhancement is an issue of great concern.

Propagation of whistler waves in the magnetosphere has been studied extensively.[1] Such waves can be excited during lightning events[2,3] or launched into the magnetosphere by the ground[4] or space-based VLF transmitters. Whistler waves can be ducted in an *L*-shell of the magnetosphere to continuously interact with the energetic electrons trapped in the same *L*-shell. The motions of very energetic electrons are adversely affected by the wave fields which, in essence, scatter some that are trapped in the radiation belts into a loss cone.[1] Induced electron precipitation[3,5–8] by whistler waves has been observed.

The Doppler shifted electron cyclotron resonance interaction[9–14] has been suggested to be responsible for the correlation of the presence of magnetospheric whistler waves and the electron precipitation events. However, the simultaneous observation of precipitation events at geomagnetic conjugate regions in both hemispheres due to a single lightning flash[3] requires a different theoretical interpretation. While the numerical results[14] show that the electron cyclotron resonance interaction is indeed effective in scattering very energetic electrons into the loss cone, in particular, those electrons with their initial pitch angles close to the loss cone angle, the percentage of the total number of very energetic electrons which can be resonant with the wave at a given frequency is small. It suggests that this process (relying on cyclotron resonance interaction) would require that the wave have a broad frequency spectrum, so that a substantial fraction of the very energetic electrons can be precipitated.

In our early work,[15–17] we have shown that the trajectories of trapped very energetic electrons in the presence of a whistler wave can become chaotic. Once it occurs, many of them will wander into the loss cone. This chaotic scattering process can be an effective approach for controlling the population of very energetic electrons in the radiation belts. There is a threshold field requirement for the commencement of chaos in the electron trajectories. Favorably, it has been observed[18–20] that trapped energetic (keV) electrons in the magnetosphere can significantly amplify whistler waves. Another supporting evidence of whistler wave amplification by energetic electrons is the natural event of chorus[21,22] occurring in the inner magnetosphere. The energy transfer is also through the electron cyclotron resonance interaction.[23–26] This suggests an optimal approach, which applies the chaotic scattering process under a double resonance situation, for the control of the population of very energetic (MeV) electrons trapped in the magnetosphere. This approach is first to amplify the incident whistler waves by the trapped keV electrons (having a loss-cone velocity distribution) in the radiation belts; the amplified whistler waves then scatter the trapped MeV electrons, which are also in cyclotron resonance with the wave, into the loss cone. In the previous work,[27] 5 keV electrons were considered for wave amplification and double resonance conditions could not be satisfied simultaneously. Thus, the chaotic scattering process did not invoke cyclotron resonance; consequently, its threshold wave field was shown to be quite high.

In the present work, we show that when 50 keV electrons are considered for the wave amplification, wave can also be in cyclotron resonance with 1.5 MeV electrons during the pitch angle scattering. Thus the threshold for the commencement of chaos is reduced considerably. In Sec. II, this amplification process is analyzed numerically. Amplification gain vs the energetic electron density is evaluated. The peak intensity, the modulation period, and the gain of the amplified wave vs the incident wave intensity are also determined. In Sec. III, pitch angle scattering of very energetic electrons by a whistler wave under double resonances is studied. We first use the double resonance conditions to determine the initial parameters of those very energetic (1.5 MeV) electrons which are resonantly interacting with the wave. These parameters are then used in the evaluation of pitch angle scattering. Summary and conclusions are presented in Sec. IV.

II. LOSS-CONE NEGATIVE MASS INSTABILITY: AMPLIFICATION OF WHISTLER WAVES

The magnetospheric electron plasma consists of three components: (1) cold background, (2) energetic (in the keV range) electrons, and (3) very energetic (in the MeV range) electrons. The background electron plasma is nonrelativistic ($\gamma \sim 1$) and the momentum P of the electron has a loss cone distribution, which can be modeled as

$$F_b(P_\perp,P_z) = n_b(\pi^{-3/2}/j!)(\Delta P_{jb})^{-(2j+3)}P_\perp^{2j} \times \exp[-(P_\perp^2+P_z^2)/\Delta P_{jb}^2], \quad (1)$$

where j is the loss cone index, and the loss cone angle $\varphi_L = \tan^{-1}\sqrt{j}$; $\Delta P_{jb}=[mT_{eb}/(1/2+j/3)]^{1/2}$; subscript b denotes "background."

The background electron plasma determines the propagation characteristics of the whistler wave, which has the dispersion relation

$$\omega = \Omega_0 c^2k^2/\omega_{pb}^2, \quad (2)$$

where $\omega_{pb}=(4\pi n_b e^2/m)^{1/2}$; n_b is the background cold electron density; $\Omega_0=eB_0m/c$ is the nonrelativistic electron cyclotron frequency.

Energetic electrons carry weak relativistic effects ($\gamma_0 \sim 1.1$, i.e., 50 keV electrons) and have significant density n_ε to amplify whistler waves. The electron's momentum also has a loss cone distribution given by

$$\begin{aligned} f_\varepsilon(P_\perp,P_z) &= 2\pi P_\perp F_\varepsilon(P_\perp,P_z) \\ &= n_\varepsilon(2\pi^{-1/2}/j!)(\Delta P_{j\varepsilon})^{-(2j+3)}P_\perp^{2j+1} \\ &\quad \times \exp[-(P_\perp^2+P_z^2)/\Delta P_{j\varepsilon}^2] \\ &= n_\varepsilon f_{\varepsilon\perp}(P_\perp) f_{\varepsilon z}(P_z), \end{aligned} \quad (3)$$

where $\Delta P_{j\varepsilon}=[mT_{e\varepsilon}/(1/2+j/3)]^{1/2}$; $T_{e\varepsilon} \gg T_{eb}$; subscript ε stands for "energetic."

In the following, an instability process to amplify whistler waves by energetic electrons having a loss cone velocity distribution in the magnetosphere is studied. The loss cone distribution makes this component potentially unstable to microinstability. The relativistic effect, through the Doppler shifted cyclotron resonance interaction provides an effective channel for the energy exchange between the electrons and the waves. The resonance condition is given by

$$\omega = \Omega_0/\gamma + \mathbf{k}\cdot\mathbf{v}, \quad (4)$$

where $\omega < \Omega_0$ for the whistler waves; ω and Ω_0 are the wave frequency and the nonrelativistic electron cyclotron frequency, and γ is the relativistic factor of the electron. For small γ, i.e., $\omega < \Omega_0/\gamma$, the resonant electrons are moving oppositely to the wave propagation direction.

The relativistic effect provides nonlinearity in wave-electron interaction, which results in bunching those electrons in near cyclotron resonant interaction with the wave. The electron bunching provides a positive feedback to excite the loss-cone negative mass instability for the wave amplification.[28] In the present work we consider only wave amplification in time, a local uniform magnetic field will be assumed. The resonant trajectory equations of a single electron in the whistler wave fields and in a uniform background magnetic field $\hat{\mathbf{z}}\, B_0$ are given by[29]

$$\mathrm{d}\alpha/\mathrm{d}t = -(k/\Omega_0)(eE_0/m)(1+kv_z/\omega)\cos\phi, \quad (5)$$

$$\mathrm{d}\phi/\mathrm{d}t = -\Delta\omega + (k/\Omega_0)(eE_0/m)(1+kv_z/\omega)\alpha^{-1}\sin\phi, \quad (6)$$

$$\mathrm{d}v_z/\mathrm{d}t = (\Omega_0/k)(eE_0/mc^2)(\alpha/\gamma^2)(kc^2/\omega+v_z)\cos\phi, \quad (7)$$

$$\mathrm{d}\gamma/\mathrm{d}t = -(\Omega_0/k)(eE_0/mc^2)(\alpha/\gamma)\cos\phi, \quad (8)$$

where $\alpha=kv_\perp/\Omega_0$ and $\phi=kz_0+[\theta_0+\varphi(t)]-\int_0^t\Delta\omega'\mathrm{d}t'$; $\Delta\omega =\Delta\omega_0+\Omega_0(\gamma-\gamma_0)/\gamma\gamma_0+k(v_z-v_{z0})$, $\Delta\omega_0=\omega-\omega_0$, and $\omega_0 =\Omega_0/\gamma_0-kv_{z0}$; $\theta_0=\tan^{-1}(v_{y0}/v_{x0})$, φ accounts for the phase shift in the electron gyration due to interaction with the wave fields, γ_0 is the initial relativistic factor of the resonant elec-

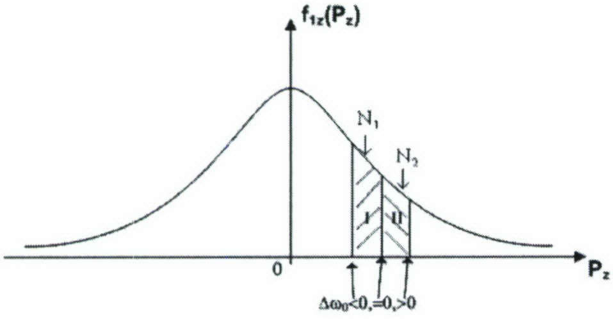

FIG. 1. Momentum distribution of energetic electrons and the regions close to cyclotron resonant interaction with the wave.

trons. The ratio of (7) and (8) leads to an invariant relation $\gamma(kc^2/\omega+v_z)=\text{const}=\gamma_0(kc^2/\omega+v_{z0})$, which is used to obtain $\Delta\omega=\Delta\omega_0-(k^2c^2/\omega-\omega_0)(\gamma-\gamma_0)/\gamma$. It is noted that $k^2c^2/\omega-\omega_0>0$ for whistler waves considered in the present work, i.e., $\Delta\omega$ increases as electrons lose energy to the wave and vice versa.

Velocity of the electron plasma in the magnetosphere has a loss-cone distribution. Only a small portion of the total number of electrons, e.g., in the two shade regions in the velocity distribution shown in Fig. 1, is near the resonant interaction with the wave. The wave is experiencing cyclotron damping to the electrons which are initially at exact cyclotron resonance with the wave, i.e., $\Delta\omega_0=0$. However, the wave can exchange energy with other electrons having a mismatch frequency $\Delta\omega_0$ slightly different from zero. Depending on the initial phases of those electrons, the interaction can cause them either to gain energy from, or lose energy to, the wave, initially.

In region I, $\Delta\omega_{01}<0$; electrons reduce their mismatch frequencies in the interaction as losing energy to the wave ($\gamma-\gamma_0<0$); it results in the increase of the interaction period of losing energy to the wave. In contrast, as electrons gain energy from the wave ($\gamma-\gamma_0>0$), their mismatch frequencies increase and interaction periods of gaining energy from the wave decrease. On average, those electrons with $\Delta\omega_{01}<0$ lose energy to the wave. For those electrons in region II with $\Delta\omega_0=\Delta\omega_{02}>0$, the above dynamic interaction process is reversed, and on average, those electrons gain energy from the wave. Using the definition $\Delta\omega_0=\omega-\omega_0=\omega-\Omega_0/\gamma_0+kv_{z0}$, $\Delta\omega_{01}<\Delta\omega_{02}$ leads to $v_{z01}<v_{z02}$, which indicates that there are more electrons in the $\Delta\omega_{01}<0$ region than in the $\Delta\omega_{02}>0$ region (i.e., $N_1>N_2$ as shown in Fig. 1). Overall, the wave gains energy from electrons and thus it is amplified.

The amplification is the result of the collective effect in the electron-wave resonance interaction. Thus the single particle resonant trajectory equations (5)–(8) are averaged over the electron's random phase angle (with respect to the wave field) to obtain the governing equations for the induced current densities in the energetic electron plasma. The phase average is based on a similar procedure as that presented in Ref. 27. After phase average, (8) is converted to an energy conservation equation, which can be integrated to obtain the relation $\langle\Delta\omega\rangle_{\varepsilon 1}=\Delta\omega_{01}+(p\omega_b^2/\Omega_0)[\varepsilon_0(1+\varepsilon_r)/2\gamma_0\Delta n_\varepsilon mc^2]\times[E_0^2(t)-E_0^2(0)]$, where $\langle\,\rangle_{\varepsilon 1}$ represents an average over the initial random phases of those energetic electrons in region I of Fig. 1; $\Delta n_\varepsilon=N_1-N_2\cong j^{-1/2}n_\varepsilon[\mathrm{j}^{-1/2}f'_{\varepsilon\perp}(P_{\perp 1})f_{\varepsilon z}(P_{z1})-f_{\varepsilon\perp}(P_{\perp 1})f'_{\varepsilon z}(P_{z1})](2\gamma_1 m\Delta\omega_0/k)^3$ is the net density of energetic electrons transferring energy to the wave through the resonant interaction with the wave; $\epsilon_r=1+\omega_{pb}^2/\omega\Omega_0=(kc/\omega)^2$ is the dielectric function of background plasma responding to the whistler wave.

The phase averaged equations derived from the three first order differential equations (5)–(7) are then combined with the wave equation, which is a second order differential equation, to yield a single differential-integral equation,[27] in essence a fifth order ordinary differential equation

$$
\begin{aligned}
&[d_t^3+2\Gamma d_t^2+(\Delta\underline{\omega}^2+\underline{C})d_t]E_0\\
&\quad=a_0\{1+(N_\varepsilon/\Delta n_\varepsilon)(\omega_{pb}^2/\Omega_0\Delta\omega_{01})\\
&\qquad\times[\varepsilon_0(1+\varepsilon_r)/4\gamma_0\Delta n_\varepsilon mc^2][E_0^2(t)-E_0^2(0)]\}\\
&\qquad\times\int_0^t E_0(t')\cos\langle\Delta\phi(t-t')\rangle_{\varepsilon 1}\mathrm{d}t', \qquad (9)
\end{aligned}
$$

where $\Gamma\cong-[3\varepsilon_r/8(1+\varepsilon_r)](b_1\alpha_0/\gamma_0^2)(A_0E_0I_c/\omega)$ and

$$
\begin{aligned}
\Delta\omega^2&=\Delta\omega_{01}^2+\alpha_0^2\{s^2[(1+v_{z0}\omega/kc^2)+3k(\omega/c)^2(1+kv_{z0}/\omega)\\
&\quad-(9/4)\alpha_0^2k(\omega/c)^4(1+kc_{z0}/\omega)^2]E_0^2\\
&\quad-\omega\Delta\omega_0(1+kv_{z0}/\omega)\}\\
&\quad-(3b_1\alpha_0/4\gamma_0^2-2\Delta\omega_0^2/sE_0^2)A_0I_sE_0;
\end{aligned}
$$

$$
s=(k/\Omega_0)(e/m)(1+kv_{z0}/\omega)\alpha_0^{-1},
$$

$$
I_c=\int_0^t E_0(t')\cos\langle\Delta\phi(t-t')\rangle_{\varepsilon 1}\mathrm{d}t'
$$

and

$$
I_s=\int_0^t E_0(t')\sin\langle\Delta\phi(t-t')\rangle_{\varepsilon 1}\mathrm{d}t';
$$

$$
C=[\Delta\omega_{p\varepsilon}^2/(1+\varepsilon_r)\gamma_0][(1+kv_{z0}/\omega)-1/2(\Omega_0\alpha_0/\gamma_0 kc)^2],
$$

$$
a_0=-[\Delta\omega_{p\varepsilon}^2\Omega_0^2\Delta\omega_{01}\alpha_0^2/(1+\varepsilon_r)\gamma_0^3\omega],
$$

and

$$
\Delta\omega_{p\varepsilon}^2=\Delta n_\varepsilon e^2/m\varepsilon_0.
$$

Equation (9) governs the temporal evolution of the field amplitude of the whistler wave interacting resonantly with this particular group of trapped electrons (in the shade regions of Fig. 1) in the radiation belts. Equation (9) is linearized to obtain the relation $|\Delta\omega_{01}|^3=\Delta\omega_{p\varepsilon}^2\Omega_0^2\alpha_0^2/16(1+\varepsilon_r)\gamma_0^3\omega$, which is used to determine $|\Delta\omega_{01}|$. We next analyze this equation numerically to determine the conditions for the amplification of whistler waves.

For numerical analysis, (9) is normalized to a dimensionless form by introducing

$$
X=[\varepsilon_0(\varepsilon_\mathrm{r}-1)|\Delta\omega_{01}|/4\omega\gamma_0\Delta n_\varepsilon mc^2]^{1/2}E_0(t),
$$

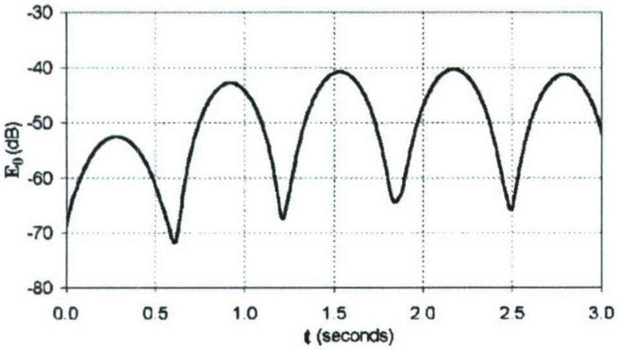

FIG. 2. Temporal evolution of the amplitude of a whistler wave traversing the magnetosphere.

$$\xi = |\Delta\omega_{01}|t,$$

and

$$g = (N_\varepsilon/\Delta n_\varepsilon)(\varepsilon_r+1)kc\omega\omega_{pb}^2/\Omega_0(\varepsilon_r-1)\Delta\omega_{01}^2$$
$$= (N_\varepsilon/\Delta n_\varepsilon)(\varepsilon_r+1)(\omega/\Delta\omega_{01})^2.$$

The normalized equation has the form

$$[d_\xi^3 - 48Xf_c d_\xi^2 + (1+bX^2+48f_s X)d_\xi]X = 16[1-g(X^2-X_0^2)]f_c \quad (10)$$

where $X_0=X(0)$; $f_c=\int_0^\xi X(\xi')\cos\varphi(\xi,\xi')d\xi'$ and $f_s=-\int_0^\xi X(\xi')\sin\varphi(\xi,\xi')d\xi'$; $\varphi(\xi,\xi')=\int_{\xi'}^{\xi}\{1-2g[X^2(\xi'')-X_0^2]\}d\xi''$;

$$b = [4\Delta\omega_{p\varepsilon}^2k^2c^2/\gamma_0(\varepsilon_r-1)|\Delta\omega_{01}|^3\omega][(1+v_{z0}\omega/kc^2)$$
$$+3(\omega\Omega_0/\gamma_0k^2c^2)-(9/4)\alpha_0^2(\omega\Omega_0/\gamma_0k^2c^2)^2]$$
$$\cong 192[(\varepsilon_r+1)/(\varepsilon_r-1)](\omega\gamma_0/\alpha_0^2\Omega_0);$$

b and g are constant coefficients; (10) is subjected to the initial conditions: $X(0)=X_0$, $d_\xi X(0)=\sqrt{3}X_0$, $d_\xi^2X(0)=3X_0$.

In the present work, we consider the resonant energetic electrons of 50 keV, which is different from the previous case[27] for 5 keV. This significant increase of the energy of the resonant energetic electrons is necessary in order to achieve the double resonances as being shown in the next section. Having the background parameters, $b=1440$ and $g=2\times10^7$ (i.e., $\varepsilon_r\sim4$ and $|\Delta\omega_{01}|/\omega\sim5\times10^{-4}$), and setting $X_0=3.58\times10^{-4}$, (10) is solved by an ordinary differential equation (ODE) solver. The result is presented in Fig. 2, showing the temporal evolution of the field amplitude $E_0(t)$. The dB scale plot in Fig. 2 is for a direct comparison with the early Siple experimental result.[18–20] In Siple experiments,[18] injected Siple signals of 3 kHz, propagating along the $L\cong4$ shell, were often amplified by 10–30 dB and oscillated in time. The numerical result presented in Fig. 2 also shows that whistler wave amplitude is amplified more than 25 dB by trapped relativistic electrons through the loss-cone negative mass instability and also oscillates in time in a similar fashion, a good agreement with the experimental observation.

The dependence of the wave amplitude gain G on the density of energetic electrons is through the parameter g in

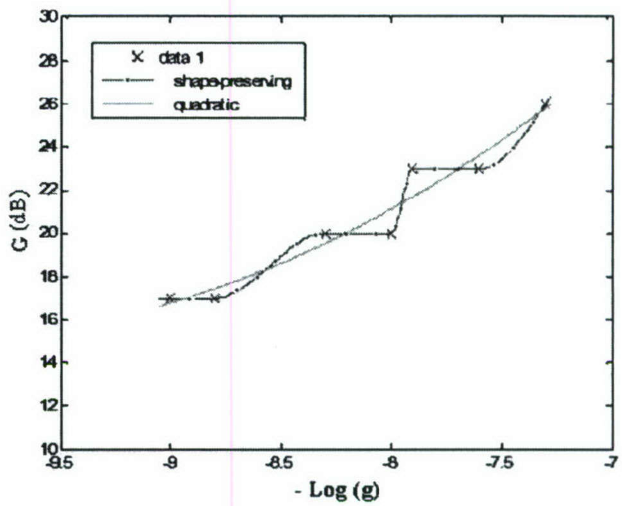

FIG. 3. Amplitude gain G of the amplified whistler wave as a function of the density of energetic electrons ($\propto -\log(g)$).

(10). Since g is inversely proportional to $(\Delta n_\varepsilon/N_\varepsilon)\times(\Delta\omega_{01}/\omega)^2\propto\Delta n_\varepsilon^{5/3}/N_\varepsilon$, the gain G in dB is plotted against $-\log(g)$ in Fig. 3, where $G=20\log(E_{0m}/E_{in})$ and E_{0m} and E_{in} are the maximum amplitude of the amplified wave and the amplitude of the incident wave, respectively. As shown, the gain G increases (nearly linearly) with the density of energetic electrons, as expected.

It is noted that the field amplitude E_{in} of the incident whistler wave in Fig. 2 is rather low (to be consistent with that of previous experiments); however, in the practical application for achieving significant electron precipitation, the incident wave field has to increase considerably. Therefore, it is important to realize how the gain G varies with the incident wave intensity for a fixed background condition. This is exemplified by considering a case with the following background conditions: $\Omega_0/\omega=2$, $\gamma_0=1.1$, and $\varepsilon_r=11$; $|\Delta\omega_{01}|/\omega\sim5\times10^{-4}$ and $v/c=0.417$; setting $v_\perp/c=0.32$, leads to $\alpha_0=0.358$, $b=456$, and $g=3.2\times10^6$. The peak intensity $I_m(\propto E_{0m}^2)$ of the amplified wave versus the intensity I_{in} $(\propto E_{in}^2)$ of the incident wave is presented in Fig. 4. As shown, the rate of increase of I_m decreases as I_{in} increases. In other words, G (I_{in}) is a decreasing function of I_{in}, which is presented in Fig. 5. It shows that with a 30 dB increase of I_{in}, G is reduced by about 10 dB. The process of wave amplification is via phase bunching of gyrating electrons as the positive feedback of wave-electron interaction. However, when the phase bunching exceeds 180°, the interaction turns to debunch electrons. Thus the amplitude of the wave oscillates during the continuous interaction. The oscillation period is the sum of the bunching and debunching times, which decrease as the wave intensity increases. Consequently, the modulation period T (in seconds) of the wave intensity is expected to decrease with I_{in} because I_m increases with I_{in}. This dependence T (I_{in}) is also presented in Fig. 4.

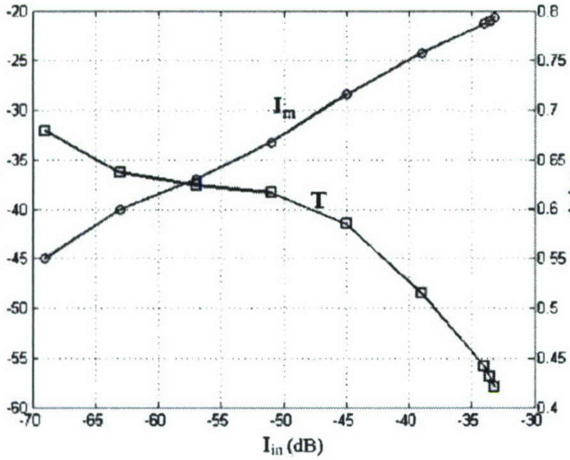

FIG. 4. The dependency of the peak intensity I_m and the modulation period T of an amplified wave on the intensity I_{in} of the incident wave.

III. DOUBLE CYCLOTRON RESONANCES FOR EFFECTIVE PRECIPITATION OF VERY ENERGETIC ELECTRONS BY WHISTLER WAVES

Whistler waves can cause the trajectories of trapped energetic electrons in the radiation belts to be chaotic, leading to a scattering of very energetic (MeV) electrons into the loss cone. However, this chaotic scattering process requires a threshold wave field for the commencement of chaos in the electron trajectories. The amplification process considered in the prior section is aimed at reducing the power requirement of the incident whistler waves. In addition, cyclotron resonance is found to be the optimal condition to achieve the minimum threshold power requirement. To take the advantage of both effects, a double resonance condition has to be satisfied, namely, the wave can be simultaneously in cyclotron resonance with energetic (a few tens of keV level) electrons as well as with very energetic electrons.

A wave resonant with MeV electrons satisfies a resonance condition, similar to that given by (4),

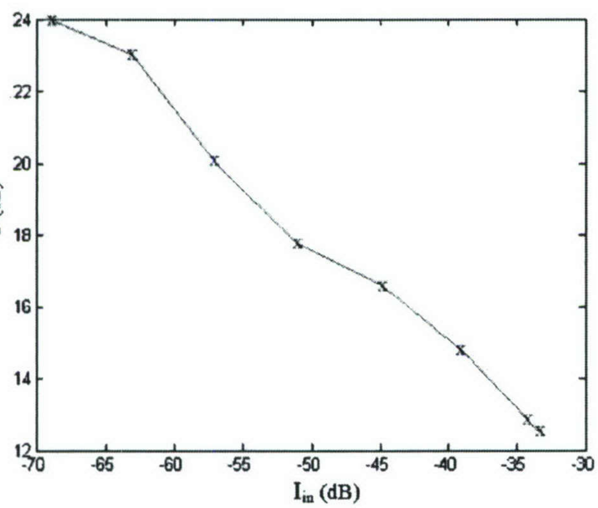

FIG. 5. Amplitude gain G of the amplified whistler wave as a function of the intensity of the incident whistler wave.

$$\omega = \Omega_0/\gamma_2 + kv_{z2}. \tag{11}$$

To show that a double resonance situation is possible, we first express the resonance condition in a general form $\omega = \Omega_0/\gamma + kP_z/\gamma m$, where $\gamma=(1+P_\perp^2/m^2c^2+P_z^2/m^2c^2)^{1/2}$, $P_\perp = \gamma m v_\perp$, and $P_z=\gamma m v_z$. Because of the γ dependence, this condition leads to a quadratic equation for P_z as $AP_z^2 + 2BP_z + C = 0$, where $A=(1-\omega^2/k^2c^2)$, $B=m\Omega_0/k$, and $C=(m/k)^2[\Omega_0^2-\omega^2(1+P_\perp^2/m^2c^2)]$. This quadratic equation has two real solutions $P_z=[-B\pm(B^2-AC)^{1/2}]/A$, subject to the condition $B^2 \geqslant AC$. The double solutions suggest that the wave can be simultaneously resonant with two different groups of electrons. The coefficients A and C of the quadratic equation are positive because $\omega/kc<1$ and $\Omega_0/\gamma_{1,2}>\omega$ are considered; thus both P_z are negative, i.e., the two groups of electrons, which can resonantly interact with the wave, move opposite to the propagation direction of the wave.

We now find the initial conditions of electrons such that (4) and (11) can be satisfied simultaneously. The relationships and notations to be applied are first introduced as follows: $\Omega_0 \propto L^{-3}$; $p\omega_b \propto L^{-3/2}$; $\omega_B \propto (1-\gamma_2^{-2})^{-1/2}\Omega_0/L$ is the bounce frequency of trapped very energetic electrons; $n \cong (p\omega_b^2/\omega\Omega_0)^{1/2}$ is the index of refraction of the background cold plasma on the whistler wave; L is the number of earth radii, i.e., L value of a magnetic flux tube; $\theta_1=\tan^{-1}(j)^{1/2}$ is the initial pitch angle of the energetic electrons which contribute to wave amplification, thus $\cos\theta_1 = 1/(1+j)^{1/2}$; $\gamma_1 \sim 1.1$ is the relativistic factor of the energetic electrons; θ_2 is the initial pitch angle of trapped very energetic electrons which are being precipitated; the normalized phase velocity of the wave $V=v_p/c=(\Omega_0/\omega_p)\xi^{-1/2}$, where $\xi=\Omega_0/\omega$.

With the aid of some known background parameters: $n_b \sim 280\ \mathrm{cm}^{-3}$ at $L=4.9$, $B_0 \sim 0.25$ Gauss at $L=1$, and $\omega_B/\Omega_0 \sim (1/3)\times 10^{-2}$ for $\gamma_2=3$ and $L=2$, we can obtain the functional dependence of the background parameters on L as $p\omega_b = 2\pi \times 1.63\times 10^6/L^{3/2}$, $\Omega_0 = 2\pi\times 7\times 10^5/L^3$, and $\omega_B/\Omega_0 = (\omega_B/\omega)/(\Omega_0/\omega) = 7.07\times 10^{-3}(1-\gamma_2^{-2})^{-1/2}/L$.

The resonance conditions (4) and (11) and the dispersion relation lead to

$$\begin{aligned}\Omega_0/\omega &= \gamma_2 + n(\gamma_2^2-1)^{1/2}\cos\theta_2 = \xi \\ &= \gamma_1 + n(\gamma_1^2-1)^{1/2}\cos\theta_1,\end{aligned} \tag{12}$$

$$n = 2.33L^{3/2}\xi^{1/2} = 2.33L^{3/2}[\gamma_2 + n(\gamma_2^2-1)^{1/2}\cos\theta_2]^{1/2}. \tag{13}$$

Equations (12) and (13) are solved to yield

$$\begin{aligned}\cos\theta_2 = 1/2[(\gamma_1^2-1)/(\gamma_2^2-1)]^{1/2} \times \{(\gamma_2/\gamma_1+1)\cos\theta_1 \\ -(\gamma_2/\gamma_1-1)[\cos^2\theta_1 + 4\gamma_1/5.43L^3(\gamma_1^2-1)]^{1/2}\}.\end{aligned} \tag{14}$$

Thus, (12) and (13) become

$$\begin{aligned}\Omega_0/\omega \rightarrow \Omega_0 = &[\gamma_2(\gamma_1^2-1)^{1/2}\cos\theta_1 \\ &- \gamma_1(\gamma_2^2-1)^{1/2}\cos\theta_2]/[(\gamma_2^2-1)^{1/2}\cos\theta_1 \\ &- (\gamma_2^2-1)^{1/2}\cos\theta_2]n \\ = &(\gamma_2-\gamma_1)/[(\gamma_1^2-1)^{1/2}\cos\theta_1 \\ &- (\gamma_2^2-1)^{1/2}\cos\theta_2]\end{aligned}$$

and the normalized (to ω) bounce frequency of the trapped very energetic electrons and the normalized (to c) phase velocity of the wave are given by $\omega_B=7.07\times10^{-3}(1-\gamma_2^{-2})^{-1/2}\Omega_0/L$ and $V=0.432L^{-3/2}\Omega_0^{-1/2}$.

The electron trajectory equations are similar to the set of equations used to derive the resonant trajectory equations (5)–(8), except an equivalent (restoring) force to simulate the magnetic mirror effect is also included in the equations. After a canonical transformation to a new set of canonical coordinates, the normalized electron trajectory equations for the normalized canonical coordinates (P,Q,z,p_z) in the whistler wave fields are given by[17]

$$dP/dt = -\Omega_0(\Omega_0/\gamma_2-1)Q + (\Omega_0\Omega_1/\gamma_2)\sin z, \tag{15}$$

$$dQ/dt = (\Omega_0/\gamma_2-1)(P/\Omega_0) + (\Omega_1/\gamma_2)\cos z, \tag{16}$$

$$dz/dt = p_z/\gamma_2, \tag{17}$$

$$dp_z/dt = -\omega_B^2 z + (\Omega_1/\gamma_2)(P\sin z + \Omega_0 Q\cos z), \tag{18}$$

where $\gamma_2=k(\omega/c)[k(c/\omega)^2+P^2+p_z^2+\Omega_0^2Q^2+\Omega_1^2+2\Omega_1(P\cos z-\Omega_0 Q\sin z)]^{1/2}$; $\Omega_1=eB_1/m$ and B_1 is the amplitude of the wave magnetic field. In terms of the canonical coordinates, the new Hamiltonian is found to be time independent, i.e., a constant of motion. In essence, (15)–(18) describe trajectories in a three-dimensional phase space. It is also noted that if ω_B is set to zero, (15), (16), and (18) can be combined to obtain an additional constant of motion, $\frac{1}{2}(P^2+\Omega_0^2Q^2)-\Omega_0 p_z=\text{const}$. This constant reduces the system to two autonomous equations and the trajectory will have regular behavior in a two-dimensional phase space. In other words, the bouncing motion (i.e., the magnetic mirror effect) is essential in inducing chaos in the electron trajectory. This is because bounce causes the phase relationship between the wave and the electron to change abruptly, so the electron trajectory can evolve from a regular behavior to a chaotic one. However, the chaoticity of the system is not very sensitive to the value of the bounce frequency, as long as it is not too small.[17]

We now consider the $\gamma_2=4$ case that precipitates 1.5 MeV electrons. From (14), $\theta_2=86.5°$. We then have $\Omega_0=8.6038$, $V=0.0517$, and $\omega_b=2.945\times10^{-2}$. Equations (15)–(18) are now integrated numerically to evaluate the pitch angle scattering, resulting from the wave-electron resonance interaction. Presented in Fig. 6 is a result that double resonance condition is satisfied. Thus the result presented in Fig. 6 is different from those presented in Fig. 5 of Ref. 16 and in Fig. 4 of Ref. 17. As shown, when the wave magnetic field B_1 increases to $7\times(1.16\times10^{-4}\,B_0)$, large pitch angle scattering occurs. In other words, with wave magnetic field amplitude of $0.007/8.6038=0.0008=0.08\%$ of the background magnetic field, electron is scattered to a pitch angle $<50°$, less than the loss cone angle. Further increase of the wave magnetic field B_1 to exceed $7\times(1.16\times10^{-4}\,B_0)$, i.e., $\Omega_1>0.007$, the numerical results confirm that large pitch-angle scattering persists. This is an example that a whistler wave, with proper parameters, can be amplified by the electrons having energies in tens of keV ranges and simultaneously scatters MeV electrons, both processes via the cyclotron resonance interaction. Comparing the result presented in Fig. 6 with those of the nonresonant cases presented in Refs. 16, 17, and 27, the cyclotron resonance interaction reduces the threshold wave magnetic field amplitude for achieving precipitation by more than an order of magnitude. However, the required interaction time is increased by more than two orders of magnitude.

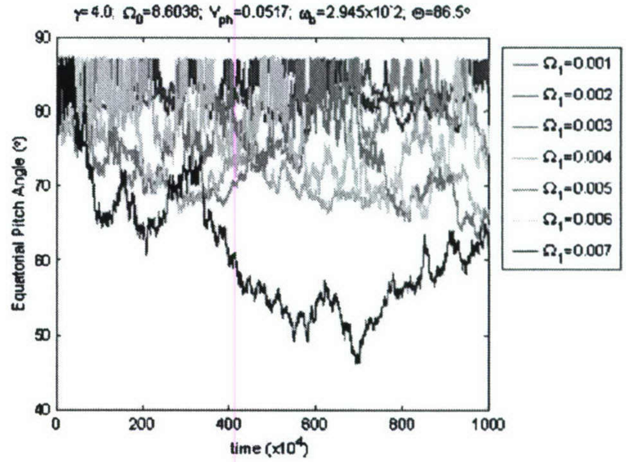

FIG. 6. (Color online) Temporal evolution of the pitch angle of a 1.5 MeV electron, which is interacting with a whistler wave at Doppler shifted cyclotron resonance. The wave magnetic field B_1 increases consecutively from 1 to $7\times(1.16\times10^{-4}\,B_0)$.

IV. SUMMARY AND CONCLUSION

It is shown that energetic electrons in the radiation belts can amplify a whistler wave through a loss-cone negative mass instability. The gain factor increases nearly linearly with the density of energetic electrons. The results show that this amplification process can enhance the field intensity of a very low amplitude incident whistler wave by more than 25 dB. The theory is formulated and the numerical result is shown to agree well with the experimental result, both qualitatively and quantitatively. However, the peak amplitude of the amplified wave still depends on the amplitude of the incident wave. In order to achieve significant electron precipitation, the field amplitude has to exceed a threshold level. Hence, the amplitude of the incident wave has to increase considerably from the level of the previous Siple experiments, as well as from those of currently available VLF transmitters. It is found that this gain decreases as the incident wave intensity increases; however, it reduces to less than 10 dB for a 30 dB increase of the incident wave intensity. In other words, this amplification process will still be effective and reduce considerably the required field intensity of the incident whistler wave for the purpose of precipitating those very energetic electrons in the MeV range.

An optimal approach to reduce the population of MeV electrons trapped in the magnetosphere is suggested as follows. To precipitate those electrons with pitch angles close to the loss cone angle, the required wave field is below a threshold for the onset of chaos.[14] Small pitch angle scatterings via electron cyclotron resonance interaction will be adequate to diffuse those electrons into the loss cone.[13] However, to precipitate those electrons trapped far away from the loss cone, it will need a chaotic scattering process. In this case the wave field has to exceed a threshold for being able to cause the electron trajectories chaotic. Although the threshold field is reduced considerably at cyclotron resonance, the numerical result shows that it is still quite large. The numerical results demonstrate that a 1.5 MeV electron can be scattered from an initial pitch angle of 86.5° to a pitch angle $<50°$ by a whistler wave with the magnetic field amplitude of 0.08% of the background magnetic field. It converts to about 3100 pT at $L=2$, and 200 pT at $L=5$. Thus wave amplification is indeed needed. We have shown that the background energetic electrons (e.g., <100 keV electrons) can amplify injected whistler waves through a loss-cone negative mass instability by more than 15 dB. We have further shown that a double resonance condition can be satisfied, namely, that the wave can be cyclotron resonant with energetic (a few tens of keV level) electrons for amplification, and yet still be able to precipitate MeV electrons via effective cyclotron resonance interaction. Finally, it should be pointed out that this optimal approach, relying on electron cyclotron resonance interaction, requires that the wave have a broad frequency spectrum, so that a considerable fraction of the very energetic electrons can be precipitated simultaneously. A broadband wave can be considered as a sum of many narrow band components and different components have different central frequencies. Each narrow band wave component will interact with a distinctive group of electrons determined by the cyclotron resonance conditions. Thus the amplification and precipitation processes for each narrow band wave component will be similar to those for a monochromatic wave considered in the present work. However, when the wave is amplified to a large amplitude, the nonlinear coupling among the frequency components of the wave is likely to occur and should be addressed in the future work.

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